American Scientist
the magazine of Sigma Xi, The Scientific Research Society

The Music of Math Games

Keith Devlin

Search online for video games and apps that claim to help your children (or yourself) learn mathematics, and you will be presented with an impressively large inventory of hundreds of titles. Yet hardly any survive an initial filtering based on seven, very basic pedagogic "no-nos" that any game developer should follow if the goal is to use what is potentially an extremely powerful educational medium to help people learn math. A good math learning game or app should avoid:

- Confusing mathematics itself (which is really a way of thinking) with its representation (usually in symbols) on a flat, static surface.
- Presenting the mathematical activities as separate from the game action and game mechanics.
- Relegating the mathematics to a secondary activity, when it should be the main focus.
- Adding to the common perception that math is an obstacle that gets in the way of doing more enjoyable activities.
- Reinforcing the perception that math is built on arbitrary facts, rules and tricks that have no unified, underlying logic that makes sense.
- Encouraging students to try to answer quickly, without reflection.
- Contributing to the misunderstanding that math is so intrinsically uninteresting, it has to be sugar-coated.

Of the relatively few products that pass through this seven-grained filter—which means they probably at least don't do too much harm—the majority focus not on learning and understanding but on mastering basic skills, such as the multiplicative number bonds (or "multiplication tables"). Such games don't actually provide learning at all, but they do make good use of video game technology to take out of the classroom the acquisition of rote knowledge. This leaves the teacher more time and freedom to focus on the main goal of mathematics teaching, namely, the development of what I prefer to call "mathematical thinking."

Video games that provide good mathematics learning should look to the piano as a model

Many people have come to believe mathematics is the memorization of, and mastery at using, various formulas and symbolic procedures to solve encapsulated and essentially artificial problems. Such people typically have that impression of math because they have never been shown anything else. If mention of the word *algebra* automatically conjures up memorizing the use of the formula for solving a quadratic equation, chances are you had this kind of deficient school math education. For one thing, that's not algebra but arithmetic; for another, it's not at all representative of what algebra is, namely, thinking and reasoning about entire classes of numbers, using logic rather than arithmetic.

What's in a Game?

So how to go about designing a good video game to help students learn mathematics? The first step should be to read—several times, from cover to cover—the current "bible" on K–12 mathematics education. It is called *Adding it Up: Helping Children Learn Mathematics,* and it was published by the National Academies Press in 2001. The result of several years' work by the National Research Council's Mathematics Learning Study Committee, a blue-ribbon panel of experts assembled to carry out that crucial millennial task, this invaluable volume sets out to codify the mathematical knowledge and skills that are thought to be important in today's society. As such, it provides the best single source currently available for guidelines on good mathematics instruction.

The report's authors use the phrase *mathematical proficiency* to refer to the aggregate of mathematical knowledge, skills, developed abilities, habits of mind and attitudes that are essential ingredients for life in the 21st century. They break this aggregate down to what they describe as "five tightly interwoven" threads. The first is *conceptual understanding,* the comprehension of mathematical concepts, operations and relations. The second is *procedural fluency,* defined as skill in carrying out arithmetical procedures accurately, efficiently, flexibly and appropriately. Third is *strategic competence,* or the ability to formulate, represent and solve mathematical problems arising in real-world situations. Fourth is *adaptive reasoning*—the capacity for logical thought, reflection, explanation and justification. Finally there's *productive disposition,* a habitual inclination to see mathematics as sensible, useful and worthwhile, combined with a confidence in one's own ability to master the material.

The authors stress that it is important not to view these five goals as a checklist

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to be dealt with one by one. Rather, they are different aspects of what should be an integrated whole, with all stages of teaching focused on all five goals.

So it's not that the crucial information about mathematics learning required to design good learning video games is not available—in a single, eminently readable source—it's that few people outside the math education community have read it.

In the author's game, *Wuzzit Trouble*, the cute and fuzzy creatures must be freed from traps controlled by gearlike combination locks. Players collect keys to open the locks by solving puzzles of varying difficulty. (Image courtesy of InnerTube Games.)

Combining Skills

The majority of video games designed to provide mathematics learning fail educationally for one of two reasons: Either their designers know how to design and create video games but know little about mathematics education (in particular, how people learn mathematics) and in many cases don't seem to know what math really is, or they have a reasonable sense of mathematics and have some familiarity with the basic principles of mathematics education, but do not have sufficient experience in video game design. (Actually, the majority of math education games seem to have been created by individuals who know little more than how to code, so those games fail both educationally and as games.)

To build a successful video game requires an understanding, at a deep level, of what constitutes a game, how and why people play games, what keeps them engaged, and how they interact with the different platforms on which the game will be played. That is a lot of deep knowledge.

To build an engaging game that also supports good mathematics learning requires a whole lot more: understanding, at a deep level, what mathematics is, how and why people learn and do mathematics, how to get and keep them engaged in their learning, and how to represent the mathematics on the platform on which the game will be played. That too is a lot of deep knowledge.

In other words, designing and building a good mathematics educational video game—be it a massively multiplayer online game (MMO) or a single smartphone app—requires a team of experts from several different disciplines. That means it takes a lot of time and a substantial budget. How much? For a simple-looking, casual game that runs on an iPad, reckon nine months from start to finish and a budget of $300,000.

Following the tradition of textbook publishing, that budget figure does not include any payment to the authors who essentially create the entire pedagogic framework and content, nor the project's academic advisory board (which it should definitely have).

The Symbol Barrier

Given the effort and the expense to make a math game work, is it worth the effort? From an educational perspective, you bet it is. Though the vast majority of math video games on the market essentially capitalize on just one educationally important aspect of video games—their power to fully engage players in a single activity for long periods of time—all but a tiny number of games (fewer than 10 by my count) take advantage of another educationally powerful feature of the medium: video games' ability to overcome the *symbol barrier*.

Though the name is mine, the symbol barrier has been well known in math education circles for over 20 years and is recognized as the biggest obstacle to practical mastery of middle school math. To understand the symbol barrier and appreciate how pervasive it is, you have to question the role symbolic expressions play in mathematics.

By and large, the public identifies doing math with writing symbols, often obscure symbols. Why do they make that automatic identification? A large part of the explanation is that much of the time they spent in the school mathematics classroom was devoted to the development of correct symbolic manipulation skills, and symbol-filled books are the standard way to store and distribute mathematical knowledge. So we have gotten used to the fact that mathematics is presented to us by way of symbolic expressions.

But just how essential are those symbols? After all, until the invention of various kinds of recording devices, symbolic musical notation was the only way to store and distribute music, yet no one ever confuses music with a musical score.

Just as music is created and enjoyed within the mind, so too is mathematics created and carried out (and by many of us enjoyed) in the mind. At its heart, mathematics is a mental activity—a way of thinking—one that over several millennia of human history has proved to be highly beneficial to life and society.

In both music and mathematics, the symbols are merely static representations on a flat surface of dynamic mental processes. Just as the trained musician can look at a musical score and hear the music come alive in her or his head, so too the trained mathematician can look at a page of symbolic mathematics and have that mathematics come alive in the mind.

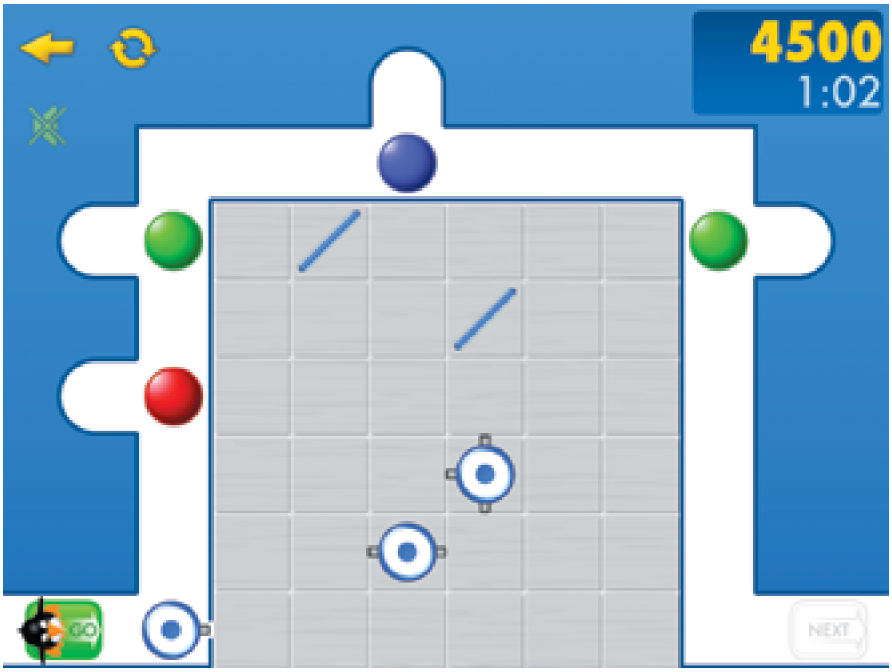
So why is it that many people believe mathematics itself is symbolic manipulation? And if the answer is that it results from our classroom experiences, why is mathematics taught that way? I can answer that second question. We teach mathematics symbolically because, for many centuries, symbolic representation has been the most effective way to record mathematics and pass on mathematical knowledge to others.

Still, given the comparison with music, can't we somehow manage to break free of that historical legacy?

Though the advanced mathematics used by scientists and engineers is intrinsically symbolic, the kind of math important to ordinary people in their lives—which I call everyday mathematics—is not, and it can be done in your head. Roughly speaking, everyday mathematics comprises counting, arithmetic, proportional reasoning, numerical estimation, elementary geometry and trigonometry, elementary algebra, basic probability and statistics, logical thinking, algorithm use, problem formation (modeling), problem solving, and sound calculator use. (Yes, even elementary algebra belongs in that list. The symbols are not essential.)

True, people sometimes scribble symbols when they do everyday math in a real-life context. But for the most part, what they write down are the facts needed to start with, perhaps the intermediate results along the way and, if they get far enough, the final answer at the end. But the doing-math part is primarily a thinking process—something that takes place mostly in your head. Even when people are asked to "show all their work," the collection of symbolic expressions that they write down is not necessarily the same as the process that goes on in their minds when they do math correctly. In fact, people can become highly skilled at doing mental math and yet be hopeless at its symbolic representations.

With everyday mathematics, the symbol barrier emerges. In their 1993 book *Street Mathematics and School Mathematics*, Terezinha Nunes, David William Carraher and Analucia Dias Schliemann describe research carried out in the street markets of Recife, Brazil, in the early 1990s. This and other studies have shown that when people are regularly faced with everyday mathematics in their daily lives, they rapidly master it to an astonishing 98 percent accuracy. Yet when faced with what are (from a mathematical perspective) the very same problems, but presented in the traditional symbols, their performance drops to a mere 35 to 40 percent accuracy.



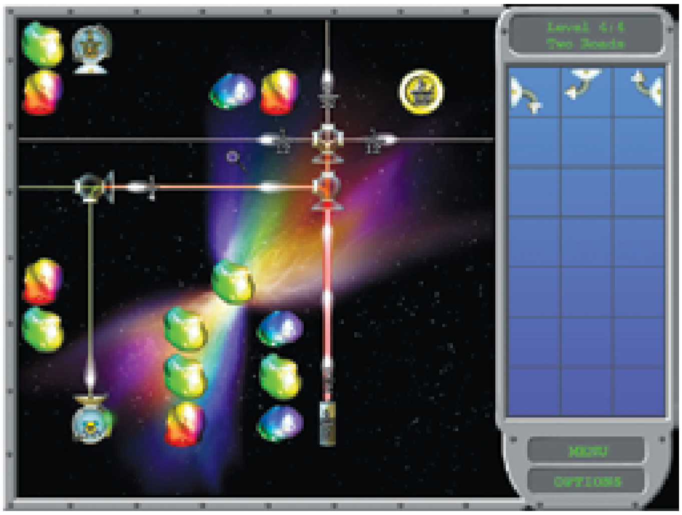
KickBox **uses a penguin character called JiJi that players must help get from one end of the corridor to the other. Players position beam-splitters and reflectors to direct lasers that knock out obstacles in JiJi's path. Solving such a puzzle provides excellent practice in mathematical thinking, completely separate from the more familiar formulas, equations and dreaded "word problems." (Image courtesy of the MIND Research Institute.)**

It simply is not the case that ordinary people cannot do everyday math. Rather, they cannot do symbolic everyday math. In fact, for most people, it's not accurate to say that the problems they are presented in paper-and-pencil format are "the same as" the ones they solve fluently in a real life setting. When you read the transcripts of the ways they solve the problems in the two settings, you realize that they are doing completely different things. Only someone who has mastery of symbolic mathematics can recognize the problems encountered in the two contexts as being "the same."

The symbol barrier is huge and pervasive. For the entire history of organized mathematics instruction, where we had no alternative to using static, symbolic expressions on flat surfaces to store and distribute mathematical knowledge, that barrier has prevented millions of people from becoming pro-

MotionMath **is a *Tetris*-inspired game that uses the motion sensors in a smartphone or tablet to allow players to tilt the screen to direct descending fractions to land on the right location on the number line. This game is an excellent introduction to fractions for younger children, as it connects the abstract concept to tactile, bodily activity. (Image courtesy of MotionMath Games.)**

ficient in a cognitive skill set of evident major importance in today's world, on a par with the ability to read and write.



In the math puzzle game *Refraction*, players learn about fractions and algebra. In this puzzle, the player has to split a laser beam a sufficient number of times to power all of the alien spaceships on the screen. The game is also designed to be modified on the fly, in an effort to capture data about what teaching methods and reward systems work best for students. (Image courtesy of the University of Washington.)

Going Beyond

With video games, we can circumvent the barrier. Because video games are dynamic, interactive and controlled by the user yet designed by the developer, they are the perfect medium for representing everyday mathematics, allowing direct access to the mathematics (bypassing the symbols) in the same direct way that a piano provides direct access to the music.

It's essentially an interface issue. Music notation provides a useful interface to music, but it takes a lot of learning to be able to use it. It's the same for mathematics notation.

The piano provides an interface to music that is native to the music, and hence far more easy and natural to use. When properly designed, video games can provide interfaces to mathematical concepts that are native to those concepts, and thus far more easy and natural to use.

Consider some of the reasons so many people are able to master the piano. You learn by doing the real thing (initially poorly, on simple tunes, but getting better over time). You use the very same instrument on Day 1 that the professionals use. You get a sense of direct involvement with the music. You get instant feedback on your performance—the piano tells you if you are wrong and how you are wrong, so you can gauge your own progress. The instructor is your guide, not an arbitrator of right or wrong. And the piano provides true *adaptive learning*.

We read a lot today about adaptive learning, as if it were some new invention made possible by digital technologies. In fact it is a proven method that goes back to the beginning of human learning.

What's more, the proponents of today's digital version have gotten it all wrong, and as a result produce grossly inferior products. They try to use artificial intelligence so an "educational delivery system" can modify the delivery based on the student's performance.

Yet tens of thousands of years of evolution have produced the most adaptive device on the planet: the human brain. Trying to design a computer system to adapt to a human's cognitive activity is like trying to build a cart that will draw a horse. Yes, it can be done, but it won't work nearly as well as building a cart that a horse can pull.

The piano metaphor can be pursued further. There's a widespread belief that you first have to master the basic skills to progress in mathematics. That's total nonsense. It's like saying you have to master musical notation and the performance of musical scales before you can start to try to play an instrument—a surefire way to put someone off music if ever there was one. Learning to play a musical instrument is much more enjoyable, and progress is much faster, if you pick up—and practice—the basic skills as you go along, as and when they become relevant and important to you. Likewise, for learning mathematics, it's not that basic skills do not have to be mastered, but rather it's how the student acquires that mastery that makes the difference.

When a student learning to play the piano is faced with a piece she or he cannot handle, the student (usually of his or her own volition) goes back and practices some more easier pieces before coming back to the harder one. Or perhaps the learner breaks the harder piece into bits, and works on each part, at first more slowly, then working up to the correct tempo. What the player does not do is go back to a simpler piano (one with fewer keys, perhaps?), nor do we design pianos that somehow become easier to play. The piano remains the same; the player adjusts (or adapts) what they do at each stage. The instrument's design allows use by anyone, from a rank beginner to a concert virtuoso.

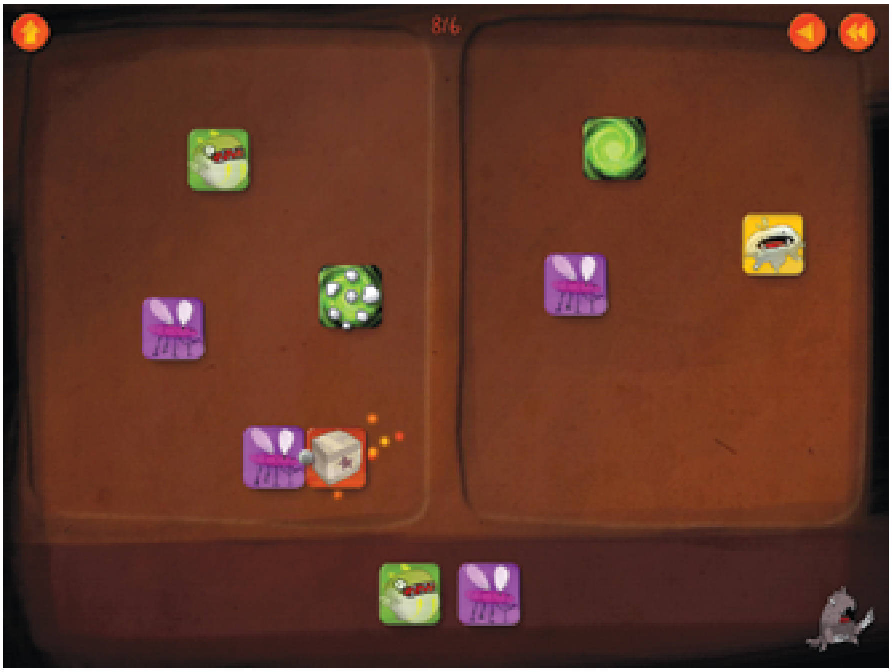
This lesson is the one we need to learn in order to design video games to facilitate good mathematics learning. For over 2,000 years, commentators have observed connections between mathematics and music. We should extend the link to music when it comes to designing video games to help students learn math, thinking of a video game as an instrument on which a person can "play" mathematics.

A Mathematical Orchestra

The one difference between music and math is that whereas a single piano can be used to play almost any tune, a video game designed to play, say, addition of fractions, probably won't be able to play multiplication of fractions. This means that the task facing the game designer is not to design one instrument but an entire orchestra.

Can this be done? Yes. I know this fact to be true because I spent almost five years working with talented and experienced game developers on a stealth project at a large video game company, trying to build such an orchestra. That particular project was eventually canceled, but not because we had not made progress—we had developed over 20 such "instruments"—but because the pace and cost of development did not fit the company's entertainment-based financial model. A small number of us from that project took all that we had learned and formed our own company, starting from scratch to build our own orchestra.

In the meantime, a few other companies have produced games that follow the same general design principles we do. Some examples include the games *MotionMath* and *MotionMath Zoom,* which use the motion sensors in a smartphone or tablet to allow players to interact directly with numbers. The puzzle game *Refraction* was produced by a group of professors and students in the Center for Game Science at the University of Washington, and was designed as a test platform that could be altered on the fly to see what teaching methods and reward systems work best for students learning topics such as fractions and algebra. *DragonBox* focuses on learning algebra in a puzzle where a dragon in a box has to be isolated on one side of the screen. *KickBox* uses physical concepts—such as positioning lasers to get rid of obstacles for the game's penguin mascot—to learn math concepts. The same producer, the MIND Research Institute, also developed *Big Seed*, a game where players have to unfold colored tiles to completely fill a space. These games all combine the elements of math learning with game play in an effective, productive fashion.



DragonBox challenges players to isolate the glittering box (containing a growling dragon) on one side of the screen. What they are doing is solving for the _x_ in an algebraic equation. But there isn't an _x_ to be seen in the early stages of the game. As the player progresses through the game, mathematical symbols start to appear, first as names for the objects, later replacing the object altogether. This game demonstrates very clearly that solving an algebraic equation is not fundamentally about manipulating abstract symbols, but is reasoning about things in the world, for which the symbols are just names. _DragonBox_ provides a more user-friendly interface to algebraic equations—but it's still algebra, and even young children can do it. (Image courtesy of We Want To Know Games.)

The game produced by my colleagues and me, because we were working in our spare time and were entirely self-funded until early last year, has taken us three years to get to the point of releasing. Available in early March, *Wuzzit Trouble* is a game where players must free the Wuzzits from the traps they've inadvertently wandered into inside a castle. Players must use puzzle-solving skills to gather keys that open the gear-like combination locks on the cages, while avoiding hazards. As additional rewards, players can give the Wuzzits treats and collect special items to show in a "trophy room."

We worked with experienced game developers to design *Wuzzit Trouble* as a game that people will want to play purely for fun, though admittedly mentally challenging, puzzle entertainment. So it looks and plays like any other good video game you can play on a smartphone or tablet. But unlike the majority of other casual games, it is built on top of sound mathematical principles, which means that anyone who plays it will be learning and practicing good mathematical thinking—much like a person playing a musical instrument for pleasure will at the same time learn about music. Our intention is to provide, separately and at a later date, suggestions to teachers and parents for how to use the game as a basis for more formal learning. *Wuzzit Trouble* might look and play like a simple arithmetic game, and indeed that is the point. But looks can be deceiving. The puzzles carry star ratings, and I have yet to achieve the maximum number of stars on some of the puzzles! (I never mastered Rachmaninov on the piano either.) The game is not designed to teach. The intention is to provide an "instrument" that, in addition to being fun to play, not only provides implicit learning but may also be used as a basis for formal learning in a scholastic setting. We learned all of these design lessons from the piano.